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Pseudoclassical model for topologically massive gauge fields

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Abstract: A pseudoclassical model for P,T-invariant system of topologically massive U(1) gauge fields is analyzed. The model demonstrates a nontrivial relationship between continuous and discrete symmetries and reveals a phenomenon of "classical quantization". It allows one to identify SU(1,1) symmetry and S(2,1) supersymmetry as hidden symmetries of the corresponding quantum system. We show this P,T-invariant quantum system realizes an irreducible representation of a non-standard super-extension of the $(2+1)$-dimensional Poincaré group.

Keywords: Field Theories in Lower Dimensions, Chern-Simons Theories, Space-Time Symmetries, Global Symmetries.

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Contents

1 Introduction

Revealing new symmetries and investigating their structures is the most powerful and productive approach in modern physics [1]. Classical particle models are useful for finding hidden properties of corresponding quantum systems and understanding their nature: efficiency of the approach, based on pseudoclassical mechanics [2] underlying the path-integral formulation of the quantum field theory with fermions [3], was recently demonstrated in the context of constructing the covariant formulation of the superstring theory [4], for which the Brink-Schwarz superparticle model [5] played very important role. In a previous paper [6] we proposed a new pseudoclassical model, whose quantum analog is parity and time-reversal conserving system of topologically massive vector U(1) gauge fields [7]. The latter may be relevant to the quantum Hall effect [8] and high-temperature superconductivity [9], that basically motivates our studies.

Here we continue investigations started with ref. [6]: The purpose of this paper is to uncover hidden symmetries of the Chern-Simons fields' system by means of the corresponding particle model. To be self-contained, we will recollect properties of our pseudoclassical construction. It contains a c-number parameter q, entering a mass term for spin variables, which displays a quantization property both at the classical and the quantum levels. The parameter does not affect discrete P and T symmetries of the classical theory, but turns out to be crucial for continuous global symmetries: there are special discrete values of q at which the pseudoclassical model has a maximal set of integrals of motion. The same values of the parameter are separated by the requirement of maximality of global symmetry of the physical state space at the quantum level. Moreover, we shall see that only at these special values discrete symmetries of the pseudoclassical model are conserved upon quantization. It is interesting to note here that Deser, Jackiw and

Templeton discovered the quantization of the dimensionless mass-coupling-constant ratio in the non-Abelian vector case having implicated symmetry arguments [7].

Analyzing algebras of the integrals of motion we shall reveal hidden SU(1,1) symmetry and S(2,1) supersymmetry of the P,T-invariant system of topologically massive vector U(1) gauge fields and show that the system realizes an irreducible representation of a non-standard super-extension of the (2+1)-dimensional Poincaré group. Though here we deal with a free theory, these results can hopefully find further development and applications in elementary particle and condensed matter physics. In this respect, it is worthwhile remembering that revealing of SU(N) symmetries in hadron and nuclear physics [10] and the discovery of supersymmetry [11] were realized first in contexts of free theories but subsequently turned out to be forming the basis for the description of real world. Note also that the first 4D group-theoretical supersymmetry constructions were motivated by certain aspects of parity in quantum field theory [12].

2 The field system and pseudoclassical model

The source-free equations for topologically massive vector U(1) gauge fields, given in terms of a self-dual free massive field theory [13], are first order differential equations $\mathcal{L}^{\epsilon}_{\mu\nu}\mathcal{F}^{\nu}_{\epsilon}=0$, where $\mathcal{L}^{\epsilon}_{\mu\nu}\equiv(i\varepsilon_{\mu\nu\lambda}P^{\lambda}+\epsilon m\eta_{\mu\nu})$, $P_\mu=-i\partial_\mu$, $\eta_{\mu\nu}=diag(-1,+1,+1)$, $\epsilon=+$ or $-$, and we normalize the totally antisymmetric tensor $\varepsilon^{\mu\nu\lambda}$ by $\varepsilon^{012}=1$. Due to the basic equations, the field $\mathcal{F}^{\mu}_{\epsilon}$ satisfies also Klein-Gordon equation $(P^2+m^2)\mathcal{F}^{\mu}_{\epsilon}=0$ and the transversality condition $P_\mu\mathcal{F}^{\mu}_{\epsilon}=0$. As a consequence, it carries massive irreducible representation of spin $s=-\epsilon 1$ of the 3D Poincaré group. Already in pioneering works [7] it was noted that topological mass terms are parity and time-reversal odd, and the full set of discrete C, P and T symmetries may be restored if one doubles the number of fields and introduces opposite sign mass terms. In the case under consideration, when taking the action

$$\mathcal{A}=\int d^3x\left(\mathcal{F}^{\mu}_{+}\mathcal{L}^{+}_{\mu\nu}\mathcal{F}^{\nu}_{+}+\mathcal{F}^{\mu}_{-}\mathcal{L}^{-}_{\mu\nu}\mathcal{F}^{\nu}_{-}\right),$$

we get P,T-invariant system of topologically massive vector U(1) gauge fields [7]. This observation plays an essential role in constructing models of high-temperature superconductors. Actually, single spin state models predict observable parity and time-reversal violation in corresponding superconductors [14], for which one still has no experimental evidence [15]. Besides, the problem of cancellation between single bare and radiatively generated Chern-Simons terms arises in the conventional models [16]. Therefore, it is desirable to have parity and time-reversal conserving system modeling high-T_c superconductors without these serious obstructions [17]. And for our work it means a signal to pay particular attention to the requirement of P,T invariance.

The pseudoclassical model in question here is given by the Lagrangian

$$L_q=\frac{1}{2e}\left(\dot{x}_\mu-\frac{i}{2}v\varepsilon_{\mu\nu\lambda}\xi^{\nu}_{a}\xi^{\lambda}_{a}\right)^2-\frac{1}{2}em^2-iqmv\xi^{\mu}_{1}\xi_{2\mu}+\frac{i}{2}\xi^{\mu}_{a}\dot{\xi}_{a\mu}, \tag{2.1}$$

where x_μ, $\mu = 0,1,2$, denote space-time coordinates of the particle, ξ_a^μ, $a = 1,2$, are real Grassmann variables forming two Lorentz vectors, e and v are even Lagrange multipliers, and q is a real c-number parameter. The Lagrangian (2.1) is invariant with respect to the discrete parity and time-reversal transformations

$$P: X^\mu \to \tilde{\varepsilon}(X^0, -X^1, X^2), \quad T: X^\mu \to \tilde{\varepsilon}(-X^0, X^1, X^2), \quad P:, T: (e,v) \to (e,-v),$$

where $X^\mu = x^\mu, \xi_1^\mu, \xi_2^\mu$, $\tilde{\varepsilon} = +1$ for x^μ and ξ_1^μ and $\tilde{\varepsilon} = -1$ for ξ_2^μ implying that ξ_2^μ is a pseudovector. We have to stress that in the classical theory parity and time-reversal invariance take place for any value of the parameter q. Nevertheless, we shall see that the case of $|q| = 2$ is particular both at the classical and the quantum levels of the theory, and that the quantization of the model (2.1) results in the P,T-invariant system of topologically massive vector U(1) gauge fields.

To find the corresponding quantum system and reveal its hidden symmetries, let us construct the Hamiltonian description of the model. The nontrivial canonical brackets following from the Lagrangian (2.1) are $\{x_\mu, p_\nu\} = \eta_{\mu\nu}$, $\{\xi_a^\mu, \xi_b^\nu\} = -i\delta_{ab}\eta_{\mu\nu}$, $\{e, p_e\} = 1$, $\{v, p_v\} = 1$, and we obtain two sets of primary, $p_e \approx 0$, $p_v \approx 0$, and secondary,

$$\phi = \frac{1}{2}(p^2 + m^2) \approx 0, \qquad \chi = \frac{i}{2}\left(\varepsilon_{\mu\nu\lambda}p^\mu \xi_a^\nu \xi_a^\lambda + 2qm\xi_1\xi_2\right) \approx 0,$$

constraints forming the trivial algebra of the first class with respect to the above brackets. As a consequence of the reparametrization invariance, the Hamiltonian of our model is a linear combination of the constraints, $H = e\phi + v\chi + \omega_1 p_e + \omega_2 p_v$, with the coefficients at the primary constraints being arbitrary functions of the evolution parameter τ. From the equations of motion,

$$\dot{p}_\mu = 0, \qquad \dot{x}_\mu = ep_\mu + \frac{i}{2}v\varepsilon_{\mu\nu\lambda}\xi_a^\nu\xi_a^\lambda, \qquad \dot{\xi}_{a\mu} = -v(\varepsilon_{\mu\nu\lambda}p^\nu\xi_a^\lambda - qm\epsilon_{ab}\xi_{b\mu}),$$

where $\epsilon_{ab} = -\epsilon_{ba}$, $\epsilon_{12} = 1$, we see that the energy-momentum vector p_μ and the total angular momentum vector $\mathcal{J}_\mu = -\varepsilon_{\mu\nu\lambda}x^\nu p^\lambda + \frac{i}{2}\varepsilon_{\mu\nu\lambda}\xi_a^\nu\xi_a^\lambda$ are integrals of motion.

To solve the equations for the spin variables ξ_a^μ, it is convenient to use complex mutually conjugate odd variables $b_\mu^\pm = \frac{1}{\sqrt{2}}(\xi_{1\mu} \pm i\xi_{2\mu})$ with nontrivial brackets $\{b_\mu^+, b_\nu^-\} = -i\eta_{\mu\nu}$. Taking into account the mass-shell constraint, we introduce the general notation $f^{(\alpha)} \equiv f^\mu e_\mu^{(\alpha)}$ for the projection of any Lorentz vector f^μ onto the complete oriented triad $e_\mu^{(\alpha)}(p)$, $\alpha = 0,1,2$, defined by the relations

$$e_\mu^{(0)} = p_\mu/\sqrt{-p^2}, \qquad e_\mu^{(\alpha)}\eta_{\alpha\beta}e_\nu^{(\beta)} = \eta_{\mu\nu}, \qquad \varepsilon_{\mu\nu\lambda}e^{(0)\mu}e^{(i)\nu}e^{(j)\lambda} = \varepsilon^{0ij}, \quad i,j = 1,2.$$

In terms of these, we find that the odd spin variables have the following evolution law:

$$b^{(0)\pm}(\tau) = e^{\mp iq\omega(\tau)}b^{(0)\pm}(0), \qquad b^{(i)\pm}(\tau) = e^{\mp iq\omega(\tau)}\left[\cos\omega(\tau)b^{(i)\pm}(0) + \varepsilon^{0ij}\sin\omega(\tau)b^{(j)\pm}(0)\right],$$

with $\omega(\tau) = m\int_0^\tau v(\tau')d\tau'$. From these solutions we immediately obtain quadratic nilpotent integrals of motion:

$$\mathcal{N}_0 = b^{(0)+}b^{(0)-}, \quad \mathcal{N}_\perp = b^{(1)+}b^{(1)-} + b^{(2)+}b^{(2)-}, \quad \mathcal{S} = i(b^{(1)+}b^{(2)-} - b^{(2)+}b^{(1)-}) \equiv \mathcal{J}^{(0)}.$$

The case of $q = 0$ is dynamically degenerated with the variables $b^{(0)\pm}$ being trivial integrals of motion, $b^{(0)\pm}(\tau) = b^{(0)\pm}(0)$. As we shall see, this special case is completely excluded on the quantum level. Further, we have the nilpotent second order quantities $B^\pm = \left(b^{(2)+}b^{(2)-} - b^{(1)+}b^{(1)-}\right) \pm i \left(b^{(2)+}b^{(1)-} + b^{(1)+}b^{(2)-}\right)$ satisfying a simple evolution law: $B^\pm(\tau) = e^{\pm 2i\omega(\tau)} B^\pm(0)$. We obtain that if and only if $|q| = 2$, there are two additional third order nilpotent integrals of motion in the model, namely

$$\mathcal{B}^\pm_+ = B^\pm b^{(0)\pm}, \quad \mathcal{B}^+_+ = (\mathcal{B}^-_+)^* \ \text{ for } q = 2, \quad \text{or} \quad \mathcal{B}^\pm_- = B^\pm b^{(0)\mp}, \quad \mathcal{B}^+_- = (\mathcal{B}^-_-)^* \ \text{ for } q = -2,$$

which are local in the evolution parameter τ quantities.

Thus, here we have observed some phenomenon of *classical quantization*: there are two special values of the parameter q, $q = \pm 2$, when, and only when, the system has additional (local in τ) nontrivial integrals of motion. These integrals are the generators of corresponding global symmetry transformations, and so, the system has maximal global symmetry at these two special values of the model parameter.

3 Quantization of the model

To quantize the model, we completely remove Lagrange multipliers and their conjugate momenta from the theory by gauge-fixing conditions $e - e_0 \approx 0$, $v - v_0 \approx 0$ for the primary constraints, where e_0 and v_0 are some constants. Upon quantization, the odd variables $b^\pm_\mu$ become the fermionic creation-annihilation operators $\widehat{b}^\pm_\mu$ having the only nonzero anticommutators $[\widehat{b}^-_\mu, \widehat{b}^+_\nu]_+ = \eta_{\mu\nu}$. Then the arbitrary quantum state can be realized over the vacuum $|0\rangle$, $\widehat{b}^-_\mu|0\rangle = 0$, $\langle 0|0\rangle = 1$:

$$\Psi(x) = \left(f(x) + \mathcal{F}^\mu(x)\widehat{b}^+_\mu + \frac{1}{2!}\varepsilon_{\mu\nu\lambda}\tilde{\mathcal{F}}^\mu(x)\widehat{b}^{+\nu}\widehat{b}^{+\lambda} + \frac{1}{3!}\tilde{f}(x)\varepsilon_{\mu\nu\lambda}\widehat{b}^{+\mu}\widehat{b}^{+\nu}\widehat{b}^{+\lambda}\right)|0\rangle.$$

The coefficients of this expansion are some square-integrable functions of the space-time coordinates. The quantum parity and time-reversal transformations are generated by the antiunitary operators

$$U_P = V^0_+ V^1_- V^2_+, \quad U_T = V^0_- V^1_+ V^2_+, \quad \text{with} \quad V^\mu_\pm = \widehat{b}^{+\mu} \pm \widehat{b}^{-\mu},$$

as follows,

$$P, T: \Psi(x) \to \Psi'(x'_{P,T}) = U_{P,T}\Psi(x), \qquad x'^\mu_P = (x^0, -x^1, x^2), \quad x'^\mu_T = (-x^0, x^1, x^2).$$

In correspondence with classical relations, we have

$$U_P\widehat{b}^\pm_{0,2}U_P^{-1} = \widehat{b}^\mp_{0,2}, \quad U_P\widehat{b}^\pm_1 U_P^{-1} = -\widehat{b}^\mp_1, \qquad U_T\widehat{b}^\pm_{1,2}U_T^{-1} = \widehat{b}^\mp_{1,2}, \quad U_T\widehat{b}^\pm_0 U_T^{-1} = -\widehat{b}^\mp_0.$$

While acting on the general state $\Psi(x)$ these operators induce mutual transformation of scalar, $f(x) \leftrightarrow \tilde{f}(x)$, and vector, $\mathcal{F}^\mu(x) \leftrightarrow \tilde{\mathcal{F}}^\mu(x)$, fields.

The physical states should be singled out by the quantum analogs of the remaining first class constraints, $(P^2 + m^2)\Psi = 0$ and $\widehat{\chi}\Psi = 0$, where we assume that $P_\mu = -i\partial_\mu$.

JHEP02(1998)015

Note that the nilpotent constraint χ admits no, even local, gauge condition, and so, the respective sector of the phase space can be quantized by the Dirac method only [18]. Let us fix in the quantum operator $\widehat{\chi}$ the same ordering as in the corresponding classical constraint. This gives

$$\widehat{\chi} = i\varepsilon_{\mu\nu\lambda}P^{\mu}\widehat{b}^{+\nu}\widehat{b}^{-\lambda} - qm(\widehat{b}^{+}_{\mu}\widehat{b}^{-\mu} - 3/2).$$

As a consequence of the quantum constraints, we find that $f(x) = \tilde{f}(x) = 0$, whereas for the fields $\mathcal{F}_{\mu}(x)$ and $\tilde{\mathcal{F}}_{\mu}(x)$ we get the equations

$$i\varepsilon_{\mu\nu\lambda}P^{\nu}\mathcal{F}^{\lambda} - \frac{1}{2}qm\mathcal{F}_{\mu} = 0, \qquad i\varepsilon_{\mu\nu\lambda}P^{\nu}\tilde{\mathcal{F}}^{\lambda} + \frac{1}{2}qm\tilde{\mathcal{F}}_{\mu} = 0, \tag{3.1}$$

and $(P^2 + m^2)\mathcal{F}_{\mu} = (P^2 + m^2)\tilde{\mathcal{F}}_{\mu} = 0$. Due to the linear equations (3.1) we have also $P_{\mu}\mathcal{F}^{\mu} = P_{\mu}\tilde{\mathcal{F}}^{\mu} = 0$ and $(P^2 + \frac{1}{4}q^2m^2)\mathcal{F}_{\mu} = (P^2 + \frac{1}{4}q^2m^2)\tilde{\mathcal{F}}_{\mu} = 0$. Therefore, the quantum constraints are consistent, and so, have nontrivial solutions if and only if $|q| = 2$. We have arrived at the same quantization condition which was obtained in the classical theory.

Putting $q = \epsilon 2$, $\epsilon = +$ or $-$, we finally see that the field $\mathcal{F}^{\mu}$ can be identified with the topologically massive vector U(1) gauge field $\mathcal{F}^{\mu}_{\epsilon}$, whereas the field $\tilde{\mathcal{F}}^{\mu}$ coincides with $\mathcal{F}^{\mu}_{-\epsilon}$. This gives us the desirable P,T-invariant system [7]. Let us note here that the latter can be reformulated in terms of the gauge fields through the duality relation $\varepsilon_{\mu\nu\lambda}\mathcal{F}^{\lambda}_{\epsilon} = F^{\epsilon}_{\mu\nu} = \partial_{\mu}A^{\epsilon}_{\nu} - \partial_{\nu}A^{\epsilon}_{\mu}$. In this case the corresponding basic equations are of the second order, and can thus be compared with equations of motion for another P and T conserving system – gauge-non-invariant massive model. This one, the three-dimensional Proca theory, describes causally propagating massive field excitations of spin polarizations $+1$ and -1. So, the *kinematical* contents of the gauge-invariant and non-invariant cases are identical [7, 19]. However, our pseudoclassical model has led exactly to topologically massive gauge fields. The difference between these systems may appear *dynamically*, when the vector fields interact with matter fields [20].

If we choose another ordering prescription for the quantum counterpart of the constraint function χ, we would have the same operator but with the constant term $-3/2$ changed for $\alpha - 3/2$, where the constant α specifies the ordering [21]. As a result, we would find that for $\alpha \neq 0, +3/2, -3/2$ under appropriate choice of the parameter q (note in this case $|q| \neq 2$) we have as a solution of the quantum constraints only one field $\mathcal{F}^{\mu}_{-}$ or $\mathcal{F}^{\mu}_{+}$ satisfying the corresponding linear differential equation. This would lead to the violation of the P and T symmetries at the quantum level. For $\alpha = +3/2$ (or $q = 0$) or $\alpha = -3/2$ the physical states are respectively described by one scalar field $f(x)$ or $\tilde{f}(x)$, and for both these cases the discrete symmetries are broken.

We see that the same values of the parameter q, $q = \pm 2$, which we have separated classically, turn out to be also special quantum mechanically: for these the number of physical states is maximal, so that the maximal global symmetry group can be realized on the physical state space, and only at $q = \pm 2$ parity and time-reversal symmetries are conserved. This result indicates a profound relationship of discrete and continuous global symmetries.

4 Revealing dynamical (super)symmetries

In what follows, we put $q=2$, the case $q=-2$ can be achieved by obvious changes. To deal with the field system obtained upon quantization of the pseudoclassical model (2.1), let us consider average value of the constraint operator $\hat{\chi}$ over an arbitrary state, $\langle\hat{\chi}\rangle=\Psi^\dagger(x)\hat{\chi}\Psi(x)$. We find that on the trivial equations of motion for unphysical scalar fields, $f(x)=\tilde{f}(x)=0$, the space-time integral of this quantity, modified on account of indefinite metric of the state space, coincides with the action $\mathcal{A}$ for P,T-invariant system of topologically massive vector U(1) gauge fields. Switching to convenient matrix notations with Pauli matrices we can write this as follows:

$$\int d^3x\langle\langle\hat{\chi}\rangle\rangle=\mathcal{A}=\int d^3x\Phi^\dagger(x)\left(PJ\otimes 1+m\cdot 1\otimes\sigma_3\right)\Phi(x),$$

where we have introduced a doublet of vector fields, $\Phi=(\mathcal{F}_+,\mathcal{F}_-)$ in transposed form, redefined there the scalar product to $\langle\langle.\rangle\rangle=\Phi^\dagger.\Phi$ and used generators $(J_\mu)^\alpha{}_\beta=-i\varepsilon^\alpha{}_{\mu\beta}$ in the vector representation of the 3D Lorentz group, $[J_\mu,J_\nu]=-i\varepsilon_{\mu\nu\lambda}J^\lambda$, $J_\mu J^\mu=-2$. The procedure we have implemented in order to get the field action $\mathcal{A}$ is reminiscent of the idea suggested in [22] for constructing a string field theory action and subsequently developed in [23].

Continuous global symmetries of this quantum system are generated by average values of the quantum counterpart of the third order nilpotent integrals of motion taking place at $q=2$. They are given by the expression $\mathcal{Q}^\pm=-\frac{1}{2}\langle\langle\hat{\mathcal{B}}^\mp_+\rangle\rangle=\Phi^\dagger(x)Q^\pm\Phi(x)$, where the quantum mechanical nilpotent operators

$$Q^\pm=\frac{1}{4i}J^2_\pm\otimes\sigma_\pm$$

realize mutual transformation of the physical states of spin $+1$ and -1. Here we use the notation $\sigma_\pm=\sigma_1\pm i\sigma_2$ and $J_\pm=J^{(1)}\pm iJ^{(2)}$, $J^{(\alpha)}=J^\mu e^{(\alpha)}_\mu$. Commutation relation of these operators is

$$[Q^+,Q^-]_-=\frac{1}{2}(S-\Pi),$$

where $S=J^{(0)}\otimes 1$ and $\Pi=J^{(0)}J^{(0)}\otimes\sigma_3$. S is the spin operator corresponding to the average value of the quantum counterpart of the integral $\mathcal{S}$, and Π is the operator associated with the projector onto the physical spin ±1 states, $(1+2\widehat{\mathcal{N}}_0)\widehat{\mathcal{N}}_\perp(2-\widehat{\mathcal{N}}_\perp)$. We have also

$$[Q^\pm,S]_-=\pm2Q^\pm,\qquad [Q^\pm,\Pi]_-=\mp2Q^\pm.$$

The operators $Q^\pm$ reproduce the algebra of the quantum mechanical counterpart of the integrals $\mathcal{B}^\mp_+$. When considering the following linear combinations

$$Q_0=\frac{1}{4}(S-\Pi),\qquad Q_1=\frac{1}{2}(Q^++Q^-),\qquad Q_2=\frac{i}{2}(Q^+-Q^-),$$

we obtain that the quantum physical operators Q_α, $\alpha=0,1,2$, form $su(1,1)$ algebra,

$$[Q_\alpha,Q_\beta]_-=-i\varepsilon_{\alpha\beta\gamma}Q^\gamma.$$

JHEP02(1998)015

The generators Q_α and the Casimir operator $C = Q_\alpha \eta^{\alpha\beta} Q_\beta = \frac{3}{8}(S\Pi - \Pi^2)$ of this algebra form also $s(2,1)$ superalgebra [24],

$$[Q_\alpha, Q_\beta]_+ = \eta_{\alpha\beta}\frac{2}{3}C, \qquad [Q_\alpha, C]_- = 0.$$

The Casimir operator takes the value $C = -3/4$ on the physical subspace given by two square-integrable transversal vector fields $\mathcal{F}^\mu_+$, $\mathcal{F}^\mu_-$ carrying spins -1 and $+1$.

We have thus revealed hidden SU(1,1) symmetry and S(2,1) supersymmetry of the P,T-invariant quantum system of topologically massive vector U(1) gauge fields. This hidden dynamical symmetry leads to a non-standard super-extension of the (2+1)-dimensional Poincaré group. To see this, one needs a covariant form of the above algebra relations [25]. Actually, the quantities S and Π, as well as their combination Q_0, are expressed as covariant (scalar) operators, whereas Q_i are given in terms of non-covariant quantities $J^{(i)}$, $i = 1, 2$. In the covariant form we found, the hidden (super)symmetry generators are presented by a rank-2 symmetric tensor, $X_{\mu\nu} = ie^{(0)}_\mu e^{(0)}_\nu Q_0 + X^\perp_{\mu\nu}$, provided that its traceless and transversal part $X^\perp_{\mu\nu}$ interchanges spins $+1$ and -1 [25]. Together with the energy-momentum P_μ and the total angular momentum $M_\mu = -\varepsilon_{\mu\nu\lambda}x^\nu P^\lambda \cdot 1\otimes 1 + J_\mu \otimes 1$ operators, they complete the set of generators of the superextended Poincaré group ISO$(2,1|2,1)$. The Casimir operators of this supergroup are P^2 and the *superspin* $\Sigma = \frac{1}{2}(S + \Pi)$. The eigenvalues of the superspin Σ are given by the set of numbers $(-1, 0, 0, 0, 0, 1)$. Expressing the operator C through the superspin as $C = \frac{3}{4}(\Sigma^2 - J^{(0)}J^{(0)}\otimes 1)$, we gain that the physical states are the eigenstates of the superspin operator with zero eigenvalue. Hence, the one particle states of the quantum P,T-invariant system of topologically massive vector U(1) gauge fields realize an irreducible representation of the supergroup ISO$(2,1|2,1)$ labelled by the zero eigenvalue of the superspin. Similar properties have been elucidated for the double fermion system [26], which is also considered to be relevant to high-temperature superconductivity [17].

5 Concluding remarks

In conclusion, with the help of the proposed pseudoclassical model (2.1) we have seen that the P,T-invariant system of topologically massive vector U(1) gauge fields has a rich set of hidden symmetries. It seems very likely that namely such 'internal properties' are responsible for critical phenomena in the corresponding models. Of course, it is not surprising too much, since the 3D topological mass terms [7] are originated from the θ-vacuum of 4D physics [27], and reflect its rich and complicated structure. The pseudoclassical model itself turned out to be very interesting, having revealed the quantization of the parameter q and nontrivial 'superposition' of the discrete (P and T) and continuous (hidden SU(1,1) and S(2,1)) (super)symmetries. A principal question to be answered concerning these results is to understand the sense of quantum field analog of the hidden (super)symmetry generators. It is quite natural to expect that they should be generators of the corresponding field symmetry transformations [25]. One might further study the situation with P, T-invariant matter coupling and investigate what happens with the revealed hidden (super)symmetries.

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